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On the Impact of Active Covariance Matrix Adaptation in the CMA-ES With Mirrored Mutations and Small Initial Population Size on the Noiseless BBOB Testbed

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ABSTRACT

Mirrored mutations as well as active covariance matrix adaptation are two techniques that have been introduced into the well-known CMA-ES algorithm for numerical optimization. Here, we investigate the impact of active covariance matrix adaptation in the IPOP-CMA-ES with mirrored mutation and a small initial population size. Active covariance matrix adaptation improves the performance on 8 of the 24 benchmark functions of the noiseless BBOB test bed. The effect is the largest on the ill-conditioned functions with the largest improvement on the discus function where the expected runtime is more than halved. On the other hand, no statistically significant adverse effects can be observed.

Categories and Subject Descriptors

G.1.6 [Numerical Analysis]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization

1. INTRODUCTION

The covariance matrix adaptation evolution strategy (CMA-ES) is considered as one of the standard stochastic optimization algorithms for continuous domain. In recent years, several algorithmic improvements of the original version have been made. One is the idea of increasing the population size (by a factor of two) after each restart of the algorithm

(IPOP-CMA-ES, [2]). Two other improvements, are the active covariance matrix update [9] and mirrored mutations [1]. In an accompanying paper [3], it is shown that both active covariance matrix adaptation and mirrored mutations can be easily combined in the standard IPOP-CMA-ES. Here we pursue the question how strong is the influence of the active covariance matrix adaptation on the overall performance. To this end, we compare the IPOP-CMA-ES with mirrored mutations and a small initial population size with the algorithm enhanced by active covariance matrix adaptation on the noiseless BBOB test bed. We find that the additional active covariance matrix adaptation improves the algorithm significantly on about one third of all functions while no detrimental effect can be observed.

2. ALGORITHM PRESENTATION

The baseline algorithm for our comparison is the IPOP-CMA-ES with mirrored mutations and an initial population size of $\lfloor (4 + \lfloor 3 \log(D) \rfloor) / 2 \rfloor$, i.e., a population size that is half as large as in the original IPOP-CMA-ES where D is the problem dimension. The reason is that in initial experiments, mirrored mutations showed a larger impact when the population size is small [4]. We denote this baseline algorithm as CMA_{mh} ¹. On the other hand, we have the algorithm CMA_{mah} with additional active covariance matrix adaptation which is implemented according to [8].

Both algorithms are run for a maximum of $2 \cdot 10^5 \cdot D$ function evaluations and restarted up to 10 times. Besides the reduced initial population size, all parameters are set as recommended in the original CMA-ES. The MATLAB source code is version 3.54.beta.mirrors of the CMA-ES and can be found at <http://canadafrance.gforge.inria.fr/mirroring/>.

3. TIMING EXPERIMENTS

In order to see the dependency of the algorithms on the problem dimension, the requested BBOB'2012 timing experiment has been performed for the two algorithms CMA_{mh} and CMA_{mah} on an Intel Core2 Duo T9600 laptop with 2.80GHz, 4.0GB of RAM, and MATLAB R2008b on Windows Vista SP2. The algorithms have been restarted for up to $2 \cdot 10^5 N$ function evaluations until 30 seconds have

¹The index 'm' stands for mirrored mutations and the index 'h' for half the standard population size.

passed. The per-function-evaluation-runtimes were 22; 19; 12; 8.3; 5.8; 5.8 and 12 times 10^{-4} seconds for the CMA_{mh} and 21; 19; 11; 8.3; 6.1; 5.7 and 11 times 10^{-4} seconds for the CMA_{mah} in 2, 3, 5, 10, 20, 40, and 80 dimensions respectively.

4. RESULTS

Results from experiments according to [6] on the benchmark functions given in [5, 7] are presented in Figures 1, 2 and 3 and in Tables 1. The **expected running time (ERT)**, used in the figures and table, depends on a given target function value, $f_t = f_{\text{opt}} + \Delta f$, and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach f_t , summed over all trials and divided by the number of trials that actually reached f_t [6, 10]. **Statistical significance** is tested with the rank-sum test for a given target Δf_t (10^{-8} as in Figure 1) using, for each trial, either the number of needed function evaluations to reach Δf_t (inverted and multiplied by -1), or, if the target was not reached, the best Δf -value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

The main observation is that the IPOP-CMA-ES variant with additional covariance matrix adaptation is never statistically worse than the one without active covariance matrix adaptation. The strongest positive effect of the additional covariance matrix adaptation can be observed for the ill-conditioned functions (Fig. 3) where for all test functions in this class the CMA_{mah} statistically significantly outperforms the CMA_{mh} for a target of 10^{-8} (Fig. 1). The factor of improvement in terms of ERT for these functions lies between 1.3 (for the ellipsoid function) and 2.24 (for the discus function) in 40D, see Table 1. Also on the separable ellipsoid, the CMA_{mah} statistically significantly outperforms the CMA_{mh} for all tested dimensions. Also on other functions, statistically significant improvements can be observed, mainly found for larger dimensions: for 20D and 40D on f_6 , for 10D, 20D, and 40D on f_7 , for 20D on f_8 , for 20D and 40D on f_{12} , and for 40D on f_{18} . Overall, there are only two functions where the CMA_{mh} is (slightly) better in 20D for difficult targets than the CMA_{mah}. Furthermore, within the given budget, for f_3 , f_4 , f_{19} and for all weakly structured functions, unsuccessful runs occur for both algorithms with a slight advantage for the CMA_{mah} (with 6 successful runs more than the CMA_{mh}).

In summary, active covariance matrix adaptation regularly improves the performance of IPOP-CMA-ES with mirrored mutations, also when the initial population size is smaller than the standard population size.

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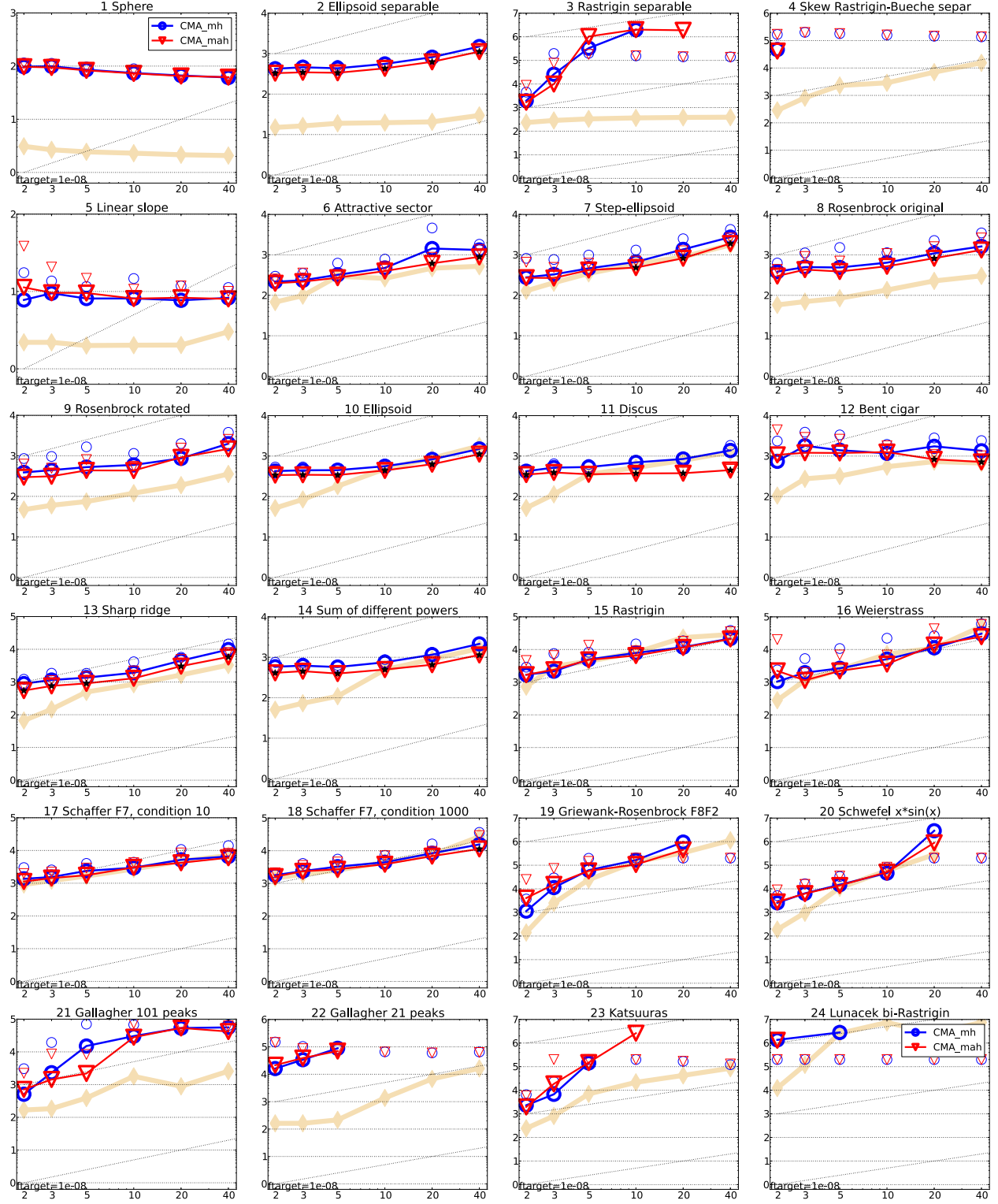


Figure 1: Expected running time (ERT in number of f -evaluations) divided by dimension for target function value 10^{-8} as \log_{10} values versus dimension. Different symbols correspond to different algorithms given in the legend of f_1 and f_{24} . Light symbols give the maximum number of function evaluations from the longest trial divided by dimension. Horizontal lines give linear scaling, slanted dotted lines give quadratic scaling. Black stars indicate statistically better result compared to all other algorithms with $p < 0.01$ and Bonferroni correction number of dimensions (six). Legend: \circ :CMA_mh, ∇ :CMA_mah.

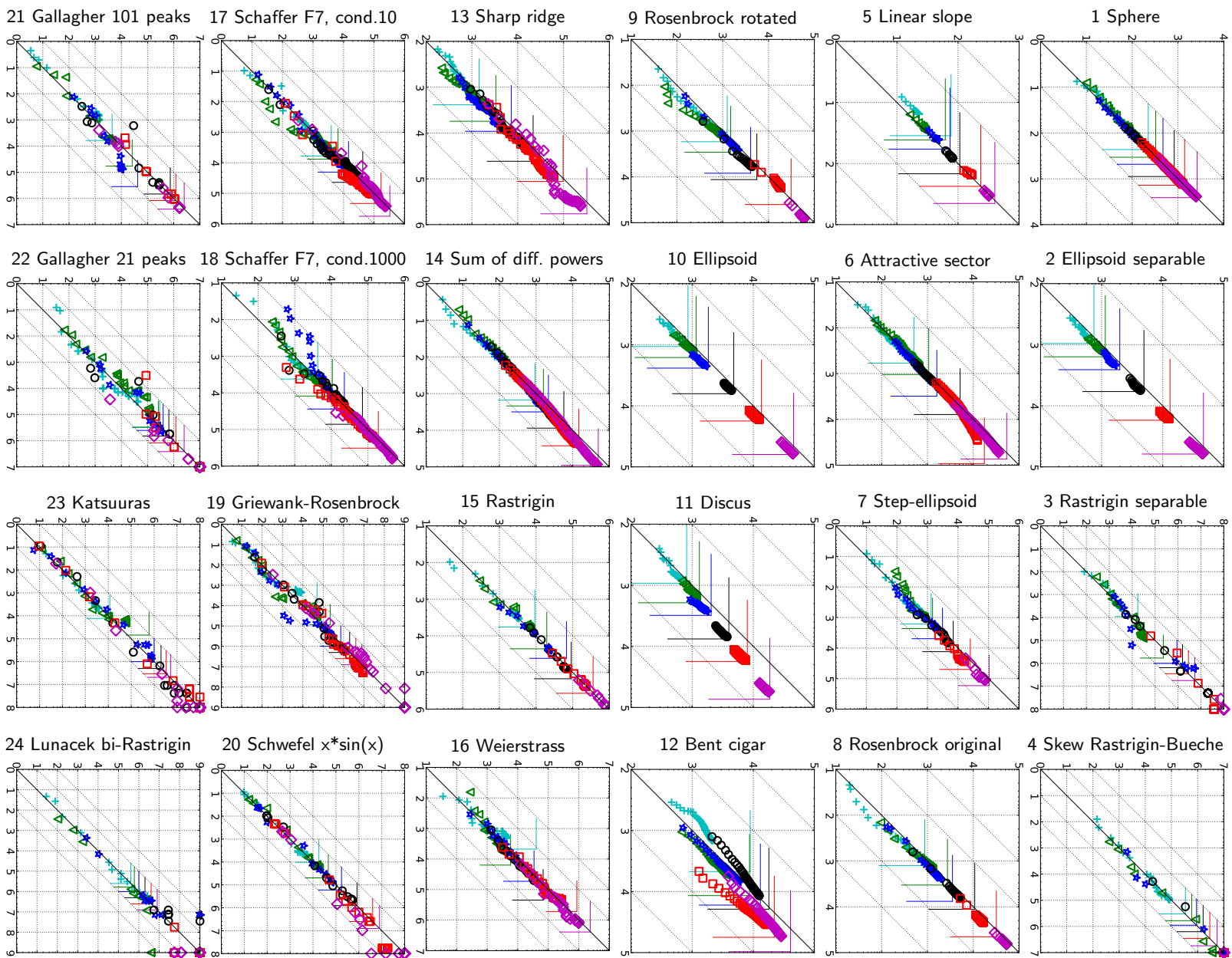


Figure 2: Expected running time (ERT in \log_{10} of number of function evaluations) of CMA-mh (x -axis) versus CMA-mh (y -axis) for 46 target values $\Delta f \in [10^{-8}, 10]$ in each dimension on functions $f_1 - f_{24}$. Markers on the upper or right edge indicate that the target value was never reached. Markers represent dimension: 2: +, 3: ∇ , 5: *, 10: o, 20: \square , 40: \diamond .

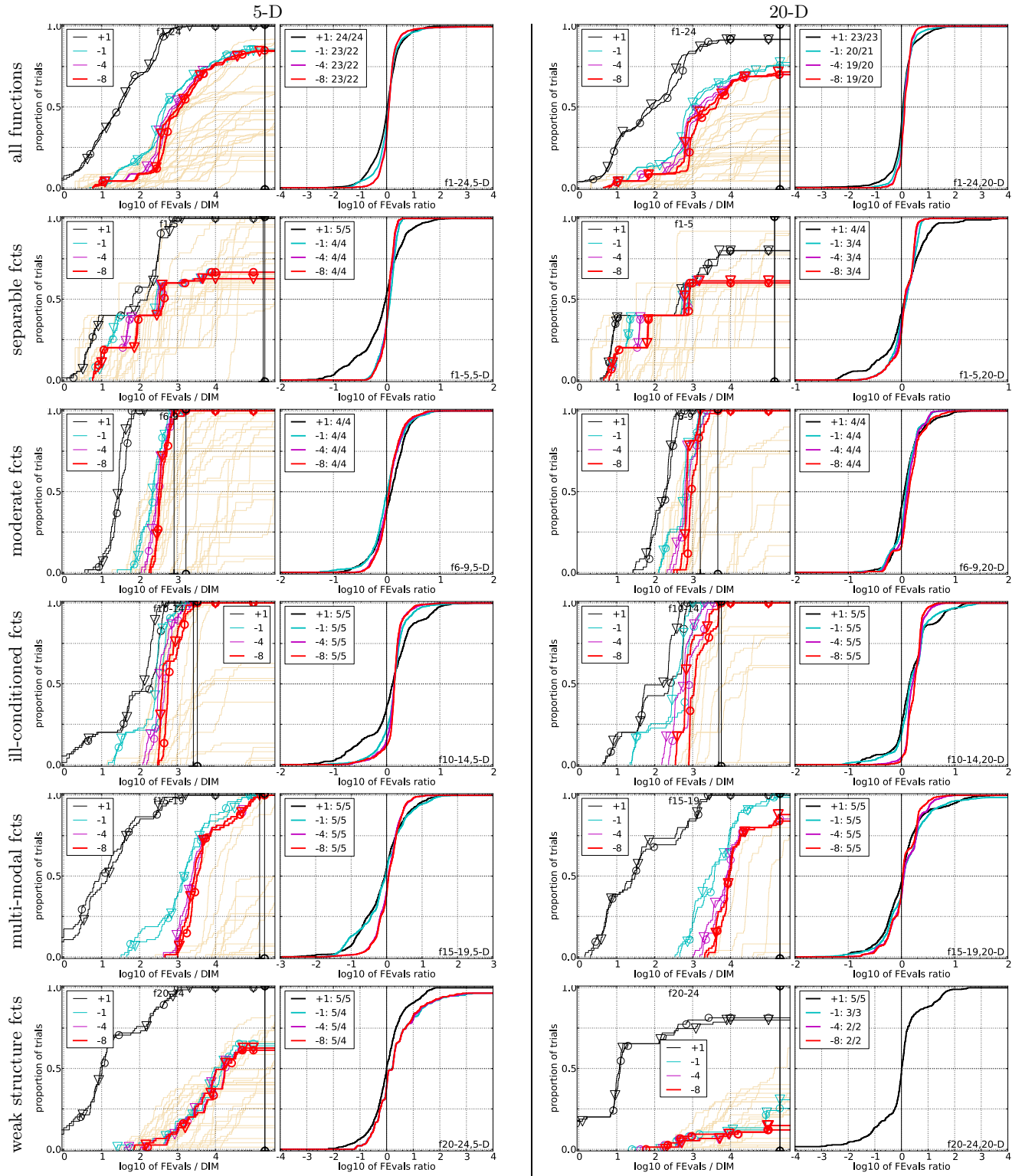


Figure 3: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right). Left sub-columns: ECDF of the number of function evaluations divided by dimension D (FEvals/ D) to reach a target value $f_{\text{opt}} + \Delta f$ with $\Delta f = 10^k$, where $k \in \{1, -1, -4, -8\}$ is given by the first value in the legend, for CMA_mh (\circ) and CMA_mah (∇). Light beige lines show the ECDF of FEvals for target value $\Delta f = 10^{-8}$ of all algorithms benchmarked during BBOB-2009. Right sub-columns: ECDF of FEval ratios of CMA_mh divided by CMA_mah, all trial pairs for each function. Pairs where one trial failed are disregarded, pairs where both trials failed are visible in the limits being > 0 or < 1 . The legends indicate the number of functions that were solved in at least one trial (CMA_mh first).

5-D

20-D

Δf	1e+1	1e-1	1e-3	1e-5	1e-7	#succ	Δf	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
f₁	11	12	12	12	12	15/15	f₁	43	43	43	43	43	15/15
1: CMA	1.8(1)	9.2(3)	16(3)	24(4)	31(4)	15/15	1: CMA	3.8(0.6)	10(1)	15(1)	22(2)	28(2)	15/15
2: CMA	1.7(1)	8.1(2)	16(2)	23(2)	30(3)	15/15	2: CMA	3.8(0.8)	10(1)	15(1)	21(1)	27(1)	15/15
f₂	83	88	90	92	94	15/15	f₂	385	387	390	391	393	15/15
1: CMA	16(5)	21(3)	21(2)	22(2)	23(2)	15/15	1: CMA	32(5)	38(2)	40(2)	41(2)	41(2)	15/15
2: CMA	13(3)	15(2)*³	16(2)*³	17(2)*³	17(2)*³	15/15	2: CMA	23(5)*³	28(2)*³	30(1)*³	31(1)*³	32(1)*³	15/15
f₃	716	1637	1646	1650	1654	15/15	f₃	5066	7635	7643	7646	7651	15/15
1: CMA	1.7(2)	1004(1285)	999(1349)	997(1410)	995(1303)	5/15	1: CMA	13(16)	∞	∞	∞	∞ 2.7e6	0/15
2: CMA	2.4(2)	3145(3852)	3129(3730)	3121(3743)	3114(3868)	2/15	2: CMA	13(7)	4950(5828)	4945(5805)	4944(5660)	4941(5716)	1/15
f₄	809	1688	1817	1886	1903	15/15	f₄	4722	7666	7700	7758	1.4e5	9/15
1: CMA	1.8(2)	∞	∞	∞	∞ 8.7e5	0/15	1: CMA	∞	∞	∞	∞	∞ 2.5e6	0/15
2: CMA	2.5(2)	∞	∞	∞	∞ 8.7e5	0/15	2: CMA	∞	∞	∞	∞	∞ 2.5e6	0/15
f₅	10	10	10	10	10	15/15	f₅	41	41	41	41	41	15/15
1: CMA	2.9(1)	4.0(1)	4.1(1)	4.1(1)	4.1(1)	15/15	1: CMA	3.2(1)	3.8(1)	3.8(1)	3.8(1)	3.8(1)	15/15
2: CMA	3.3(2)	4.7(2)	4.8(2)	4.8(2)	4.8(2)	15/15	2: CMA	3.1(0.6)	4.1(1)	4.1(1)	4.1(1)	4.1(1)	15/15
f₆	114	281	580	1038	1332	15/15	f₆	1296	3413	5220	6728	8409	15/15
1: CMA	1.8(0.9)	1.8(0.9)	1.5(0.4)	1.1(0.2)	1.1(0.2)	15/15	1: CMA	1.2(0.4)	1.1(0.3)	1.3(0.4)	1.8(0.5)	2.3(1)	15/15
2: CMA	1.7(1.0)	1.7(0.5)	1.3(0.4)	0.94(0.3)	0.92(0.2)	15/15	2: CMA	1.2(0.3)	1.0(0.2)	1.1(0.3)	1.2(0.3)*	1.3(0.3)*²	15/15
f₇	24	1171	1572	1572	1597	15/15	f₇	1351	9503	16524	16524	16969	15/15
1: CMA	4.5(5)	1.4(0.9)	1.4(0.6)	1.4(0.6)	1.4(0.7)	15/15	1: CMA	2.9(1)	2.6(1)	1.6(0.6)	1.6(0.6)	1.6(0.6)	15/15
2: CMA	3.9(3)	1.4(0.7)	1.2(0.6)	1.2(0.6)	1.2(0.6)	15/15	2: CMA	1.7(1)	1.5(0.6)*²	0.95(0.3)*²	0.95(0.3)*²	0.93(0.3)*²	15/15
f₈	73	336	391	410	422	15/15	f₈	2039	4040	4219	4371	4484	15/15
1: CMA	2.5(1)	5.1(4)	5.3(3)	5.5(3)	5.6(3)	15/15	1: CMA	3.2(1)	4.7(3)	4.8(3)	4.8(3)	4.8(3)	15/15
2: CMA	1.8(0.6)	4.0(3)	4.3(2)	4.4(2)	4.5(2)	15/15	2: CMA	2.6(0.5)	3.6(2)	3.7(2)	3.6(2)	3.6(2)	15/15
f₉	35	214	300	335	369	15/15	f₉	1716	3277	3455	3594	3727	15/15
1: CMA	5.0(2)	9.0(5)	7.6(4)	7.4(3)	7.0(3)	15/15	1: CMA	3.1(2)	4.5(0.9)	4.6(0.9)	4.6(0.8)	4.6(0.8)	15/15
2: CMA	4.4(1)	7.5(5)	6.3(3)	6.1(3)	5.8(3)	15/15	2: CMA	2.9(0.9)	4.9(3)	5.0(3)	4.9(3)	4.9(2)	15/15
f₁₀	349	574	626	829	880	15/15	f₁₀	7413	10735	14920	17073	17476	15/15
1: CMA	4.1(1)	3.3(0.3)	3.2(0.3)	2.5(0.2)	2.5(0.2)	15/15	1: CMA	1.6(0.2)	1.4(0.1)	1.1(0.0)	0.95(0.0)	0.94(0.0)	15/15
2: CMA	2.9(1)	2.4(0.3)*³	2.3(0.3)*³	1.9(0.2)*³	1.9(0.2)*³	15/15	2: CMA	1.2(0.2)*³	1.0(0.1)*³	0.79(0.1)*³↓	0.71(0.0)*³↓	0.71(0.0)*³↓	15/15
f₁₁	143	763	1177	1467	1673	15/15	f₁₁	1002	6278	9762	12285	14831	15/15
1: CMA	12(4)	3.0(0.3)	2.1(0.2)	1.7(0.1)	1.6(0.1)	15/15	1: CMA	11(0.9)	2.1(0.1)	1.5(0.1)	1.3(0.0)	1.1(0.0)	15/15
2: CMA	6.5(2)*²	1.8(0.3)*³	1.3(0.2)*³	1.1(0.1)*³	1.0(0.1)*³	15/15	2: CMA	4.8(0.5)*³	0.92(0.1)*³	0.66(0.0)*³↓	0.56(0.0)*³↓	0.49(0.0)*³↓	15/15
f₁₂	108	371	461	1303	1494	15/15	f₁₂	1042	2740	4140	12407	13827	15/15
1: CMA	8.3(10)	10(9)	10(9)	4.5(4)	4.5(4)	15/15	1: CMA	4.5(5)	6.4(5)	5.6(3)	2.4(1)	2.4(1)	15/15
2: CMA	6.5(8)	8.1(6)	8.8(5)	3.9(2)	3.8(2)	15/15	2: CMA	1.2(1)	2.6(2)*	2.5(1)*	1.1(0.4)*²	1.1(0.3)*³	15/15
f₁₃	132	250	1310	1752	2255	15/15	f₁₃	652	2751	18749	24455	30201	15/15
1: CMA	4.7(5)	8.9(2)	2.2(0.8)	2.5(1)	2.8(0.9)	15/15	1: CMA	3.7(3)	6.4(6)	1.8(0.7)	2.4(1.0)	2.7(0.7)	15/15
2: CMA	4.0(4)	5.6(3)*	1.7(0.8)	1.7(0.6)	1.6(0.5)*²	15/15	2: CMA	3.6(3)	4.2(3)	1.3(0.4)	1.4(0.6)	1.8(0.9)*	15/15
f₁₄	10	58	139	251	476	15/15	f₁₄	75	304	932	1648	15661	15/15
1: CMA	1.4(2)	2.5(0.9)	4.1(2)	6.4(0.8)	5.1(0.6)	15/15	1: CMA	2.3(1)	1.9(0.3)	2.8(0.3)	5.7(0.7)	1.2(0.1)	15/15
2: CMA	1.4(1)	2.4(0.6)	3.4(0.9)	4.1(1)*³	3.5(0.4)*³	15/15	2: CMA	2.0(0.6)	1.9(0.3)	2.3(0.3)*²	3.4(0.4)*³	0.66(0.0)*³↓	15/15
f₁₅	511	19369	20073	20769	21359	14/15	f₁₅	30378	3.1e5	3.2e5	4.5e5	4.6e5	15/15
1: CMA	3.2(3)	1.1(0.6)	1.1(0.6)	1.1(0.6)	1.1(0.6)	15/15	1: CMA	0.98(0.6)	0.66(0.3)	0.67(0.3)	0.50(0.3) [↓]	0.50(0.3) [↓]	15/15
2: CMA	2.0(2)	1.1(0.7)	1.1(0.7)	1.1(0.7)	1.1(0.7)	15/15	2: CMA	0.92(0.6)	0.64(0.3)	0.65(0.3)	0.48(0.2) [↓]	0.49(0.2) [↓]	15/15
f₁₆	120	2662	10449	11644	12095	15/15	f₁₆	1384	77015	1.9e5	2.0e5	2.2e5	15/15
1: CMA	2.9(1)	2.5(1)	1.2(1)	1.1(1.0)	1.1(1.0)	15/15	1: CMA	3.0(3)	1.1(0.7)	1.0(0.8)	1.1(0.8)	1.0(0.8)	15/15
2: CMA	2.3(1)	2.4(2)	0.94(0.7)	0.90(0.6)	0.90(0.6)	15/15	2: CMA	2.3(3)	0.90(0.6)	0.92(0.5)	1.3(1)	1.2(0.9)	15/15
f₁₇	5.2	899	3669	6351	7934	15/15	f₁₇	63	4005	30677	56288	80472	15/15
1: CMA	2.5(2)	2.6(3)	1.5(1.0)	1.4(0.6)	1.5(0.6)	15/15	1: CMA	1.8(0.8)	7.7(6)	1.4(0.8)	1.3(0.7)	1.3(0.6)	15/15
2: CMA	3.1(2)	2.1(2)	1.3(0.8)	1.1(0.5)	1.0(0.4)	15/15	2: CMA	2.0(1)	4.0(2)	0.92(0.4)	0.96(0.4)	0.94(0.3)	15/15
f₁₈	103	3968	9280	10905	12469	15/15	f₁₈	621	19561	67569	1.3e5	1.5e5	15/15
1: CMA	0.51(0.3) [↓]	1.1(1)	1.3(0.6)	1.3(0.6)	1.3(0.6)	15/15	1: CMA	3.3(8)	1.2(0.7)	1.1(0.7)	1.1(0.3)	1.1(0.3)	15/15
2: CMA	6.2(2)	1.5(1)	1.2(0.6)	1.1(0.7)	1.1(0.6)	15/15	2: CMA	0.95(0.5)	1.2(0.7)	0.79(0.3)	0.88(0.4)	0.91(0.3)	15/15
f₁₉	1	242	1.2e5	1.2e5	1.2e5	15/15	f₁₉	1	3.4e5	6.2e6	6.7e6	6.7e6	15/15
1: CMA	12(15)	421(494)	2.5(2)	2.5(2)	2.5(2)	15/15	1: CMA	82(52)	3.3(4)	1.8(2)	2.9(3)	2.8(3)	3/15
2: CMA	17(12)	518(491)	2.4(2)	2.4(2)	2.3(2)	15/15	2: CMA	95(46)	2.5(4)	0.91(0.8)	1.3(1)	1.3(1)	6/15
f₂₀	16	38111	54470	54861	55313	14/15	f₂₀	82	3.1e6	5.5e6	5.6e6	5.6e6	14/15
1: CMA	2.6(1)	1.7(2)	1.3(1)	1.3(1)	1.3(1)	15/15	1: CMA	2.6(0.8)	1.3(1)	10(12)	10(11)	10(12)	1/15
2: CMA	2.3(2)	1.6(2)	1.2(1)	1.2(1)	1.2(1)	15/15	2: CMA	2.5(0.8)	0.87(0.4)	2.4(3)	3.2(4)	3.2(4)	3/15
f₂₁	41	1674	1705	1729	1757	14/15	f₂₁	561	14103	14643	15567	17589	15/15
1: CMA	3.3(1)	40(62)	41(63)	42(70)	42(69)	14/15	1: CMA	8.7(7)	75(95)	73(118)	69(111)	61(72)	8/15
2: CMA	3.6(4)	5.7(10)	5.9(11)	6.0(11)	6.1(11)	15/15	2: CMA	25(22)	76(86)	73(84)	69(109)	61(72)	8/15
f₂₂	71	938	1008	1040	1068	14/15	f₂₂	467	23491	24948	26847	1.3e5	12/15
1: CMA	5.3(8)	288(414)	436(590)	424(548)	414(526)	7/15	1: CMA	6.9(8)	∞	∞	∞	∞ 1.1e6	0/15
2: CMA	6.3(14)	289(412)	346(449)	336(512)	329(385)	8/15	2: CMA	187(29)	∞	∞	∞	∞ 1.1e6	0/15
f₂₃	3.0	14249	31654	33030	34256	15/15	f₂₃	3.2	67457	4.9e5	8.1e5	8.4e5	15/15
1: CMA	4.5(10)	38(70)	23(32)	22(31)	21(31)	9/15	1: CMA	2.8(3)	519(602)	∞	∞	∞ 2.4e6	0/15
2: CMA	1.9(2)	52(71)	23(33)	22(31)	22(29)	9/15	2: CMA	2.9(3)	516(590)	∞	∞	∞ 2.4e6	0/15
f₂₄	1622	6.4e6	9.6e6	1.3e7	1.3e7	3/15	f₂₄	1.3e6	5.2e7	5.2e7	5.2e7	5.2e7	3/15
1: CMA	1.4(1)	2.2(2)	1.5(2)	1.1(1)	1.1(1)	1/15	1: CMA	42(42)	∞	∞	∞	∞ 4.0e6	0/15
2: CMA	1.6(1)	∞	∞	∞	∞ 1.0e6	0/15	2: CMA	42(48)	∞	∞	∞	∞ 4.0e6	0/15

Table 1: ERT in number of function evaluations divided by the best ERT measured during BBOB-2009 given in the respective first row with the central 80% range divided by two in brackets for different Δf values. #succ is the number of trials that reached the final target $f_{\text{opt}} + 10^{-8}$. 1:CMA is CMA_mh and 2:CMA is CMA_mah. Bold entries are statistically significantly better compared to the other algorithm, with $p = 0.05$ or $p = 10^{-k}$ where $k \in \{2, 3, 4, \dots\}$ is the number following the \star symbol, with Bonferroni correction of 48. A \downarrow indicates the same tested against the best BBOB-2009.